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Approximate Bayesian Computation Method: Update of the Eurocode 2 Creep model

Deterministic and Probabilistic – Rejection Algorithm Approach

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Abstract— **The deformation due to creep has an important effect on the behavior of concrete structures especially for their long term integrity. Undesirable consequences may appear in the structures due to incorrect or inaccurate prediction of creep deformation. A large database coming from international laboratories and research centers is used in order to compare the experimental results with the Eurocode 2 creep prediction. This study shows that the Eurocode 2 underestimates the important creep compliance and overestimates the small creep compliance. In order to overcome this inaccuracy, new correction coefficients are introduced to the formulas of the Eurocode 2 using an Approximate Bayesian Computation method based on the rejection algorithm.**

Keywords— **concrete, creep, Approximate Bayesian Computation method, strength, regression.**

I. Introduction

In construction, the concrete is the most used material due to its strength, durability and workability. The concrete is a material that "lives in time", it is subjected to deformations during his life. These deformations are classified into two categories: deformations due to shrinkage that are revealed without application of external loads and deformations that are manifested with application of external loads. The latter are divided into two types: instantaneous deformations that occur instantaneously when the load is applied and deformations due to creep that appear with time. The creep deformations have an important impact on the behavior, stability and durability of structures. An inaccurate creep analysis may cause excessive deflections, difficulties with closure or un-esthetic permanent deflections. Using a large experimental database provided by multinational research centers, a comparison between the experimental results and the Eurocode 2 prediction creep formulas is performed.

The Bayesian updating is an advantageous method of analyzing experimental data [1,2]. Bayesian model assessment is essential for a good evaluation of structural safety and the development of a reliability method that accounts for imperfect states of knowledge and recognizes all sources of uncertainty arising in structural problems. The Approximation Bayesian Computation method (ABC method) based on the rejection algorithm [3] will be applied in this study to introduce correction coefficients to the Eurocode 2 creep formulas.

II. Evaluation of the Eurocode 2

A. *Database*

Starting from a large database coming from international laboratories and research centers such as RILEM, LCPC, CEBTP, and from the Northwestern University's Infrastructure Technology Institute [4], a comparison is performed between the results obtained by laboratory testing and those given by the theoretical model as indicated in the Eurocode 2 – Annex B [5]. This database is composed of 1614 creep tests, each one having different properties including but not limited to water-to-cement ratio, cement type, concrete strength, effective thickness, age at loading, temperature, relative humidity, sustained stress, admixtures, etc. Since the admixtures influence the behavior of concrete, the tests with admixtures are separated from those without admixtures. Therefore, this study is based on 245 creep tests having different properties with no admixture added.

B. *Evaluation Methods*

The creep compliance $J(t,t_0)$ is the time-dependent strain per unit stress. In order to evaluate the accuracy of the Eurocode 2 creep compliance prediction on the basis of the experimental tests, four methods have been applied, in which Obs X_{ij} means the experimental creep compliance at time j of experiment i, Cal X_{ij}, the predicted creep compliance at time j of experiment i, n the total number of measurement at fixed time j of experiment i, and N the total number of experiments.

1) The Residual method

The residuals R_{ij} are calculated by the difference between the experimental compliance and the theoretical one, as given in the following equation:

$$R_{ij} = (\text{Obs } X_{ij} - \text{Cal } X_{ij}). \qquad (1)$$

The graphical representation given by the scatter plot of Eurocode 2 residuals versus experimental creep compliance is shown in Fig. 1. The scatters located near the X-axis shows that the residual is close to zero and the prediction is accurate. The scatters located below the X-axis indicate that the model overestimates the creep compliance. Contrary, the

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scatters located above the X-axis means that the model underestimates the creep compliance.

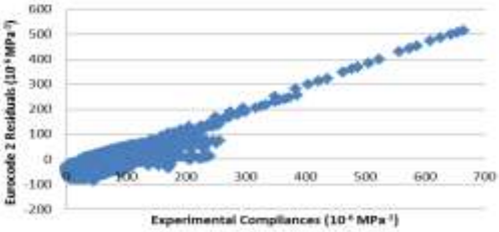

Figure 1. Eurocode 2 residuals versus experimental creep compliances ($J(t,t_0)$ in MPa^{-1}).

Fig. 1 shows clearly that before correction, the Eurocode 2 residuals increase linearly with the experimental creep compliance. This linearity is the most pronounced for the positive residuals, therefore, the more the experimental compliance increases, the more the residual increases, and then the difference between the experimental and theoretical compliance increases and the underestimation is more pronounced. In order to represent this linearity in an analytical equation, a linear regression is applied. Since Fig.1 shows that the linearity is more pronounced for $J(t,t_0) > 250\text{x}10^{-6}$ MPa^{-1}, then the linear regression will be applied for two categories, less than $250\text{x}10^{-6}$ MPa^{-1} and more than $250\text{x}10^{-6}$ MPa^{-1}.

2) The M_{CEB} method

The M_{CEB} method calculates an average gap and indicates if a model overestimates or underestimates systematically the experimental values. It may be calculated using the following formulas:

$$M_i = \frac{1}{n}\sum_{j=1}^{n}\frac{CalX_{ij}}{ObsX_{ij}}, \qquad (2)$$

$$M_{CEB} = \frac{\sum_{i=1}^{N} Mi}{N}. \qquad (3)$$

When the M_{CEB} coefficient is near 1, the values of the predicted compliance are close to the experimental results. If the M_{CEB} coefficient exceeds 1, this means that the Eurocode 2 overestimates the strains. Contrary, if the M_{CEB} coefficient is less than 1, then the Eurocode 2 underestimates the strains. In this study, by using the entire database in order to validate the time-dependent variation of the Eurocode 2, we have obtained an $M_{CEB} = 1.54$ before correction, which indicates that the Eurocode 2 overestimates the creep compliance.

3) The V_{CEB} method

The V_{CEB} method calculates an average coefficient of variation in order to evaluate a model relatively to the experimental database. By considering S_i as the standard error of ΔY_{ij} of experiment i, V_{CEB} as the average coefficient of variation, Y_i as the average value of creep of experiment i, Y_{ij} as the experimental creep at time j of experiment i, and $\Delta Y_{ij,}$ as the difference between the experimental and predicted creep compliance at time j of experiment i, then the V_{CEB} may be calculated using the following formulas:

$$Y_i = \frac{\sum_{j=1}^{n} Y_{ij}}{n}, \qquad (4)$$

$$S_i = \sqrt{\frac{1}{n-1}\times\sum_{j=1}^{n}(\Delta Y_{ij})^2}, \qquad (5)$$

$$V_i = \frac{S_i}{Y_i}\times 100, \qquad (6)$$

$$V_{CEB} = \sqrt{\frac{1}{N}\left(\sum_{i=1}^{N} V_i^2\right)}. \qquad (7)$$

Small values of V_{CEB} show that the predicted creep compliance are equal to the experimental creep compliance. In this study, we have obtained a $V_{CEB} = 197$, which indicates that before correction the Eurocode 2 does not estimate accurately the creep compliance.

4) The F_{CEB} method

The F_{CEB} method calculates the mean square error of the predicted values. By considering f_j as the difference in percentage between the predicted and experimental values and F_{CEB} as the mean square error, then the F_{CEB} may be calculated by using the following formulas:

$$f_j = \frac{(CalX_{ij} - ObsX_{ij})}{(ObsX_{ij})}\times 100, \qquad (8)$$

$$F_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n} f_j^2}, \qquad (9)$$

$$F_{CEB} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} F_i^2}. \qquad (10)$$

Similar to V_{CEB}, small values of F_{CEB} show that the predicted creep compliance are equal to the experimental creep compliance. In this study, before correction, $F_{CEB} = 414.4$, which indicates that the Eurocode 2 does not estimate accurately the creep compliance.

C. *Division of the Creep Compliance in Categories*

The division of the creep compliance in categories has been used effectively to study the impact of the range of the creep compliance. In their study [6], W. Raphael et Al. have divided the creep compliance into three categories: small creep compliance for $0 < J < 60\text{x}10^{-6}$ MPa^{-1}, medium creep compliance for $60\text{x}10^{-6}$ MPa^{-1} $< J < 120\text{x}10^{-6}$ MPa^{-1} and important creep compliance for $J > 120\text{x}10^{-6}$ MPa^{-1}. In this paper, based on the graphical representation of the residuals versus the experimental compliances, we have noticed that for the experimental compliances values less than $250\text{x}10^{-6}$ MPa^{-1}, the scatters are distributed around the X-axis with a concentration above the X-axis. But, for the experimental compliances values exceeding $250\text{x}10^{-6}$ MPa^{-1}, the scatters behave linearly and follow a line with a positive slope.

Therefore, in this study, we have subdivided the creep compliance into two categories as follows:

- Small creep compliance (SC): $J(t,t_0) < 250 \times 10^{-6}$ MPa^{-1}.
- Important creep compliance (IC): $J(t,t_0) > 250 \times 10^{-6}$ MPa^{-1}.

D. *Eurocode 2 Creep Calculation*

The creep compliance according to the Eurocode 2 is calculated based on the following equation:

$$J(t,t_0) = \frac{1}{E_{cm}(t_0)} + \frac{\phi(t,t_0)}{E_{cm28}}, \qquad (11)$$

where:

- $J(t,t_0)$ is the creep compliance,
- $E_{cm}(t_0)$ is the modulus of elasticity of concrete at the loading date t_0 in GPa,
- E_{cm28} is the modulus of elasticity of the concrete at 28 days after casting,
- $\varphi(t,t_0)$ is the creep coefficient.

The detailed calculation procedure is presented in the Eurocode 2 – Annex B [5].

E. *Approximate Bayesian Computation*

The Bayesian calibration provides an automated process for calibrating models by multiplying the expert knowledge known as *a priori* distribution by the likelihood coming from the database [1,7]. Therefore, an *a posteriori* distribution will be defined, which will be an update of the knowledge already known by using the latest database provided. Different methods of Bayesian calibration may be used. In this study, the Approximate Bayesian Computation (ABC) rejection algorithm will be applied [3]. The ABC method is applied when the likelihood function is unknown, which is our case. The Approximate Bayesian Computation rejection algorithm is based on generating a random value for each correction coefficient following an *a priori* distribution [3]. For each random variable, the updated compliance is calculated and compared to the experimental compliance. If the difference between the updated and the experimental compliance is less than the threshold, then the random variable is accepted; if not, it is rejected. Once this procedure is applied, at the end we obtain a set of correction coefficient values following a known *a posteriori* distribution or an empirical *a posteriori* distribution.

III. Results and Discussions

A. *The Residual Method Results*

In order to evaluate the best curve that fits the scatter plot of the residual values versus the experimental compliances, regressions were performed using curve expert. Fig. 2 below shows the best fit curve of the Eurocode 2 residuals versus experimental creep compliances J_{exp} scatter plot.

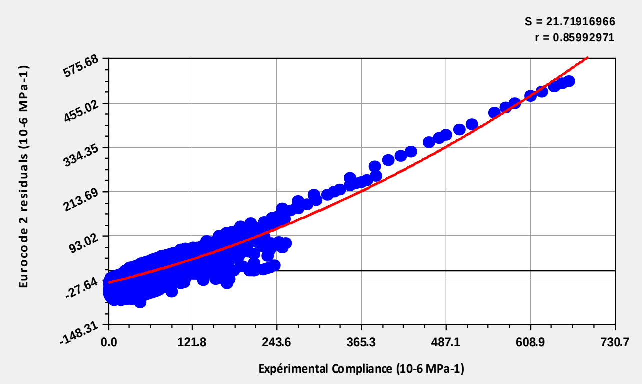

Figure 2. Eurocode 2 residuals versus experimental creep compliance quadratic fit (J in MPa^{-1}).

Fig.2 shows that the Eurocode 2 residuals versus experimental creep compliance follow a quadratic equation:

$$\text{Residual} = 6.28 \times 10^{-4} J_{exp}^{\ 2} + 0.45 J_{exp} - 34, \qquad (12)$$

with a correlation coefficient r = 0.86.

Since the residual is the difference between the experimental and theoretical values, and by considering the experimental values as the corrected theoretical values, therefore, by substituting the residual with its expression and considering the Obs $X_{ij} = J_{corr}$ and Cal $X_{ij} = J_{EC2}$, we found:

$$(6.28 \times 10^{-4})\, J_{corr}^{\ 2} - 0.55\, J_{corr} + J_{EC2} - 34 = 0, \qquad (13)$$

with $J_{corr} \times 10^{-6}$ MPa^{-1} and $J_{EC2} \times 10^{-6}$ MPa^{-1}.

Therefore, knowing J_{EC2}, which is the predicted creep compliance according to the Eurocode 2, and by substituting its value in (13), we can calculate the corrected creep compliance J_{corr} After applying (13), a comparison between the corrected creep compliance and the experimental results was performed using the evaluation methods.

TABLE I. EVALUATION METHOD VALUES BEFORE AND AFTER QUADRATIC CORRECTION

	M_{CEB}	V_{CEB}	F_{CEB}
Results Before Correction	1.54	197	414.4
Results After Quadratic Correction	1.35	148	384

Table I shows that M_{CEB}, V_{CEB} and F_{CEB} have decreased after correction. Therefore, the application of the quadratic correction results in an optimization of M_{CEB}, V_{CEB} and F_{CEB}.

As shown in Fig. 2, a linear behavior is the most pronounced for experimental compliances exceeding 250×10^{-6} MPa^{-1}. Therefore, in order to study the linear behavior of the Eurocode 2 residuals versus the creep compliance for $J(t,t_0) > 250 \times 10^{-6}$ MPa^{-1}, a division of the tests between two categories was performed. Fig. 3 and Fig. 4 show the Eurocode 2 residuals versus experimental compliances for $J(t,t_0) < 250 \times 10^{-6}$ MPa^{-1} (small creep compliance) and $J(t,t_0) > 250 \times 10^{-6}$ MPa^{-1} (important creep compliance) respectively.

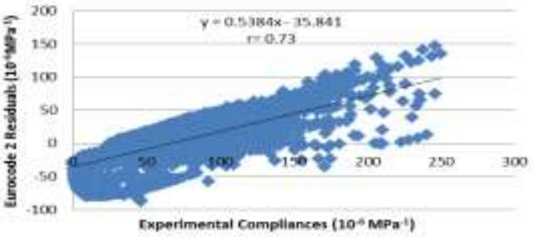

Figure 1. Eurocode 2 residuals versus experimental creep compliance linear regression for $J(t,t_0) < 250*10^{-6}$ MPa^{-1}.

The equation relating the Eurocode 2 residuals to the experimental compliance for small creep category is:

$$\text{Residual} = 0.5384 x J_{exp} – 35.841, \qquad (14)$$

with a correlation coefficient r = 0.73.

By substituting the residual with its expression and considering the Obs $X_{ij} = J_{corr}$ and Cal $X_{ij} = J_{EC2}$, we found:

$$J_{corr} = 2.16 * J_{EC2} – 77.64, \qquad (15)$$

with J_{corr} x10^{-6} MPa^{-1} and J_{EC2}x10^{-6} MPa^{-1}.

This equation is applied for $36 < J_{EC2} < 152$ (x10^{-6} MPa^{-1}). The results obtained by applying the above equation and comparing the corrected creep compliance to the experimental values are shown in Table II.

TABLE II. EVALUATION METHOD VALUES BEFORE AND AFTER LINEAR CORRECTION FOR SMALL CREEP COMPLIANCE CATEGORY

	M_{CEB}	V_{CEB}	F_{CEB}
Results Before Correction	1.54	197	414.4
Results After Linear Correction	1.37	144	386

After the linear correction, M_{CEB}, V_{CEB} and F_{CEB} have decreased from 1.54 to 1.37, from 197 to 144 and from 414.4 to 386 respectively.

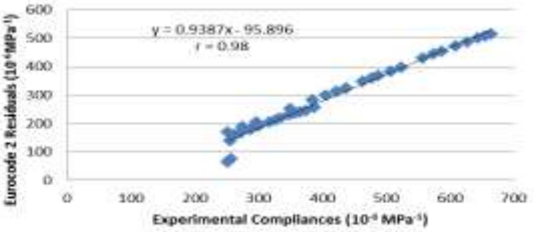

Figure 3. Eurocode 2 residuals versus experimental creep compliance linear regression for $J(t,t_0) > 250x10^{-6}$ MPa^{-1}.

The equation relating the Eurocode 2 residuals to the experimental compliance for important creep category is:

$$\text{Residual} = 0.9387 x J_{exp} – 95.896 \qquad (16)$$

This equation describes confidentially the Eurocode 2 residuals versus the experimental creep compliance since the correlation coefficient (r) is near 1 (r = 0.98). Therefore, the residual values estimated by the equation meet the exact residual values calculated from the database. By substitution we found:

$$J_{corr} = 16.31 x J_{EC2} – 1564, \qquad (17)$$

with J_{corr} x10^{-6} MPa^{-1} and J_{EC2}x10^{-6} MPa^{-1}.

This equation is applied for $J_{EC2} > 111$ (x10^{-6} MPa^{-1}).

TABLE III. EVALUATION METHOD VALUES BEFORE AND AFTER LINEAR CORRECTION FOR IMPORTANT CREEP COMPLIANCE CATEGORY

	M_{CEB}	V_{CEB}	F_{CEB}
Results Before Correction	0.5	55.7	54.4
Results After Linear Correction	0.97	36.8	41.3

Table III shows that the M_{CEB} value is almost equal to 1 after correction and the values of V_{CEB} and F_{CEB} have decreased after correction. Therefore, applying the above equation leads to a correction of the predicted values.

In order to evaluate these corrections on the entire database, a combination of the correction equations is performed as below:

$$J_{corr} = 2.16 x J_{EC2} – 77.64 \text{ for } 36 < J_{EC2} < 152 \ (x10^{-6} \text{ MPa}^{-1}) \qquad (18)$$

$$J_{corr} = 16.31 x J_{EC2} – 1564.37 \text{ for } J_{EC2} > 152 \ (x10^{-6} \text{ MPa}^{-1}) \qquad (19)$$

After applying (18) and (19), a comparison between the corrected creep compliance and the experimental results was performed using the evaluation methods.

TABLE IV. EVALUATION METHOD VALUES BEFORE AND AFTER COMBINED CORRECTION

	M_{CEB}	V_{CEB}	F_{CEB}
Results Before Correction	1.54	197	414.4
Results After Combined Correction	1.38	145	386

Table IV shows that, after the combined correction, the M_{CEB}, V_{CEB} and F_{CEB} have decreased from 1.54 to 1.38, from 197 to 145 and from 414.4 to 386 respectively.

B. *The Approximate Bayesian Computation method results*

In order to study the accuracy of the creep coefficient ratio $\frac{\varphi(t, t_0)}{E_{cm28}}$, a comparison between the experimental and theoretical values was performed for both compressive strength categories specified in the Eurocode 2, less and greater than 35 MPa.

TABLE V. THE EVALUATION METHOD RESULTS BEFORE CORRECTION FOR EACH CREEP CATEGORY: SMALL CREEP AND IMPORTANT CREEP

Categories of Creep		M_{CEB}	V_{CEB}	F_{CEB}
Creep Coefficient Ratio for $f_{cm} \leq 35$ MPa	**Small Creep**	1.67	132	393
	Important Creep	0.48	60	58
Creep Coefficient Ratio for $f_{cm} \geq 35$ MPa	**Small Creep**	1.16	125	191

Table V shows that the Eurocode 2 overestimates the small creep coefficient ratio for $f_{cm} \leq 35$ MPa and underestimates the important creep coefficient ratio for $f_{cm} \leq 35$ MPa. For $f_{cm} \geq 35$ MPa, the creep coefficient ratio is well estimated by the Eurocode 2. To overcome the difference in results between the experimental and theoretical values of the creep coefficient ratio for $f_{cm} \leq 35$ MPa, correction coefficients are introduced to the Eurocode 2 equation as follow:

$$\frac{\varphi(t,t_0)}{E_{cm28}} = O*[(1+\frac{10}{\sqrt[3]{h_0}})*\frac{(t-t_0)^{0.3}}{[\beta_H+(t-t_0)]^{0.3}} - \frac{RH*(t-t_0)^{0.3}}{10*\sqrt[3]{h_0}*[\beta_H+(t-t_0)]^{0.3}}]*\frac{1}{(0.1+t0^{0.2})}*\frac{16.8*10^{0.3}}{22000}*fcm^{-0.8*P} \quad (20)$$

With, O a global correction coefficient, and P a correction coefficient for the strength of concrete due to its important impact on the creep.

TABLE VI. THE CORRECTION COEFFICIENT VALUES FOR EACH CREEP CATEGORY

Categories of Creep		Correction Coefficients
Creep Coefficient Ratio for $f_{cm} \leq 35$ MPa	**Small Creep**	O = 2.64 P = 1.47
	Important Creep	O = 2.4 P = 0.97

Table VI summarizes the correction coefficients obtained by applying the Approximate Bayesian Computation method on the creep coefficient ratio for $f_{cm} \leq 35$ MPa. We can notice that for the small creep category, the correction coefficients are O = 2.64 and P = 1.47. As for the important creep category, the correction coefficients are O = 2.4 and P = 0.97.

TABLE VII. THE EVALUATION METHOD RESULTS BEFORE AND AFTER APPROXIMATE BAYESIAN COMPUTATION CORRECTION FOR EACH CREEP CATEGORY

Creep Coefficient Ratio for $f_{cm} \leq 35$ MPa		M_{CEB}	V_{CEB}	F_{CEB}
Small Creep	**Results Before Correction**	1.67	132	393
	Results After ABC Correction	1.23	90	272
Important Creep	**Results Before Correction**	0.48	60	58
	Results After ABC Correction	1.23	56	55

Table VII summarizes the results of M_{CEB}, V_{CEB} and F_{CEB} before and after correction of each creep category. We can notice that for the small creep category, although we have an overestimation of the creep coefficient ratio before and after correction, the mean deviation is closer to the expected value 1 by applying the correction coefficients O = 2.64 and P = 1.47. After this correction, V_{CEB} and F_{CEB} have decreased. As for the important creep coefficient, we have passed from an underestimation before correction to an overestimation after applying the correction coefficients O = 2.4 and P = 0.97. The V_{CEB} and F_{CEB} have decreased after this correction.

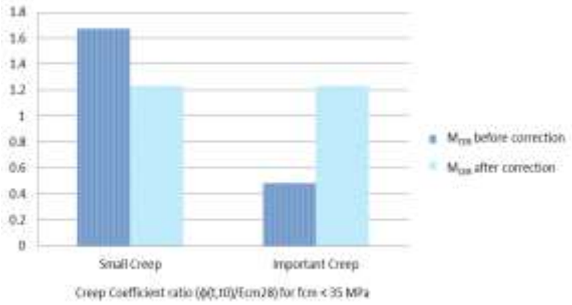

Figure 4. The M_{CEB} diagram for each creep category.

Fig. 5 shows the variation of the M_{CEB} before and after applying the Bayesian correction. For small creep category the M_{CEB} is nearer to the expected value 1 after correction. As for the important creep category, this correction leads to an overestimation instead of an underestimation of the creep coefficient ratio.

IV. Conclusions

The first step in this study consists in a comparison between theoretical results and experimental database. For $f_{cm} \leq 35$ MPa, the Eurocode 2 underestimates the important creep and overestimates the small creep. For $f_{cm} \geq 35$ MPa, the Eurocode 2 estimates accurately the creep coefficient ratio. The approach used in the design codes is clearly insufficient for practical engineering structures. The errors of ignorance and simplifications, and the measurement errors are the principal causes of code inadequacy. Two correction approaches were adopted in this study. The first correction is based on a deterministic approach; quadratic and linear equations were deducted by regressions due to the shape of the residual scatter plot versus the experimental values. This correction does not take into consideration the simplification and measurements error, therefore, the Bayesian updating approach was applied to the Eurocode 2, using Matlab and the experimental database. Correction coefficients that are implemented in the Eurocode 2 formulas were calculated for the important and small creep category for $f_{cm} \leq 35$ MPa. The Approximate Bayesian Computation rejection algorithm has proven to be an effective solution for the improvement of the creep prediction according to the Eurocode 2.

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References

[1] G. Box and G. Tiao, Bayesian Inference in Statistical Analysis, Wiley Classics Library Edition, 1992.

[2] A. Gandomi, S. Sajedi, B. Kiani, and Q. Huang, "Genetic programming for experimental big data mining: A case study on concrete creep formulation", in Automat Constr, vol. 70, Elsevier, 2016, pp. 89-97.

[3] M. Robillart "Étude de stratégies de gestion en temps réel pour des bâtiments énergétiquement performants", Thèse de doctorat de l'École Nationale Supérieure des Mines de Paris, Spécialité Énergétique, Paris, 2015.

[4] Z. P. Bažant, "Creep and shrinkage database".

[5] EN 1992-1-1: Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings, 2004.

[6] W. Raphael, A. Chateauneuf, M. Lemaire, J.L. Favre, J.A. Calgaro, and P. Peyrac, "Reliability based assessment of prestressed concrete structures subject to creep – Application to a bridge", International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP 9, Berkeley, USA, 2003.

[7] M. Riddle and R. Muehleisen, "A Guide to Bayesian Calibration of building energy models", ASHRAE/IBPSA-USA Building Simulation Conference, Atlanta, GA, USA, September 10-12, 2016.

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[The concrete is a material that "lives in time".]